Question 5 Derivatives in Portfolio Management (48 points)

a)

a1)

First, we have to compute the change in the DJ EURO STOXX 50 index, says $r_{MC,}$ corresponding to a 7.5% decline in the capital value of the managed portfolio, r_{PC} = -0.075.

From the SML equation characterizing the CAPM model, we have:

$$r_P = r_F + \beta \cdot (r_M - r_F)$$

$$r_{PC} + r_{PD} = r_F + \beta \cdot (r_{MC} + r_{MD} - r_F)$$

Where

$r_{PC} =$ capital gain rate of the portfolio

$r_{PD} =$ dividend yield of the portfolio

$r_F =$ risk-free rate

$r_{MC} =$ price index return (here, DJ EURO STOXX 50 index)

$r_{MD} =$ dividend yield of the index

$\beta =$ portfolio beta with respect to the index

In particular, we want to compute the capital index return r_{MC} over the next 12 months:

$$r_{MC} = \frac{r_{PC} + r_{PD}}{\beta} - \frac{1-\beta}{\beta} \cdot r_F - r_{MD}$$

$$r_{MC} = \frac{-0.075 + 0.02}{1.2} - \frac{1-1.2}{1.2} \cdot 0.035 - 0.03 = -0.07$$

Therefore, a 7.5% drop in the capital value corresponds to a 7% drop in the market index.

The target is to insure the managed portfolio against a 7% decline of the DJ EURO STOXX 50 over the next year with respect to a present index value of 4100, hence $K = 4100 \cdot (1 - 7\%) = 3813$.

a2)

The insurance portfolio strategy requires a long position in put options. Denoted N_{PUT} the number of put option with strike price Φ to buy, we have:

$$N_{PUT} = \beta \cdot \frac{\text{Portfolio Value}}{\text{Index Level} \cdot \text{Option Contract Size}}$$

$$N_{PUT} = 1.2 \cdot \frac{175 \cdot 10^6}{4100 \cdot 10} = 5121.95 \approx 5122$$

a3)

The main practical problems when applying static portfolio insurance are:

- Available options are sometimes only of *American type* and hence more expensive because of the right to exercise earlier than the maturity T, but in the case of portfolio insurance the investor is interested only in the value of his portfolio at expiration.
- the spread between the *time to maturity of the derivatives* available on the market *and the 'protection' horizon* may be too wide to well run the (theoretical!) strategy.
- the derivatives market may not guarantee enough *liquidity* on the desired options.
- the spread between the *theoretical floor and the strike price* available on the market may be too wide to well run the (theoretical!) strategy.
- The *beta* is an estimated quantity subject to *estimation error*.

[In general: there may not exist options on the specific stock market index, or benchmark, we require for our insurance strategy. In this exercise it is not the case, since there exist options on the DJ Euro Stoxx 50.]

b)
The put-call parity based on a stock that pays a dividend yield at rate $y = r_{MD}$ (given with continuous compounding) is:

$$S_0 \cdot e^{-r_{MD} \cdot T} + P(S_0 \cdot e^{-r_{MD} \cdot T}, T, K) = C(S_0 \cdot e^{-r_{MD} \cdot T}, T, K) + K \cdot e^{-r_F \cdot T}$$

Therefore, the portfolio insurance strategy is the following [consider that in this exercise all yields are given in simple terms]:

i) sell stock portfolio for EUR $\frac{175 \cdot 10^6}{(1+3\%)} \cong 169.90 \cdot 10^6$;
ii) buy 5,122 call options with strike price K and maturity at 1 year;
iii) invest the remainder in the risk-free activity.

Note: of course it is also correct to transform the simple rates to continuously compounded rates [by means of r = ln(1+R)] and then to use the formulas in c.c. terms.

c)
c1)
Main advantages of the use of synthetic put option:

- no liquidity risk
- allows to have exact strike and maturity date
- no basis risk

Main disadvantage: jump risk

There are two techniques to create a put option synthetically: taking a position directly in the underlying assets or trading in index futures contracts linked to the underlying assets.

Note: the following explanation is not required to get the points, and is presented only for pedagogical purposes:

A good reason to use futures contracts to create portfolio insurance instead of trading in the underlying asset is that the associated transaction costs are generally lower.

To create a synthetic put option by trades in the underlying stocks, the investor should ensure that at any given time a proportion equal to –Δ of the stocks in the managed portfolio has been sold and the proceeds invested in riskless assets, where Δ is the delta of the considered put option:

$$\Delta = e^{-r_{PD}T}[(N(d_1) - 1)].$$

As the value of the original portfolio declines, the Δ of the put becomes more negative and the proportion of the original portfolio sold must be increased. As the value of the original portfolio increases, what has been sold must be repurchased.

When using futures contacts, the original portfolio is preserved and the delta of the managed portfolio is kept in balance by shorting futures contracts for a monetary amount equivalent to the proportion of underlying assets which should be sold.

c2)

In our case, the delta of the required put option is [consider that the dividend yield is given here in simple terms]:

$$\Delta = \frac{(0.6178 - 1)}{1 + 2\%} = -0.37471$$

Therefore, 37.47% of the managed portfolio should be initially sold.

Note: of course it is also correct to transform the simple rates to continuously compounded rates [by means of r=ln(1+R)] and then to use the formulas in c.c. terms.

r = ln(1+R) => r = ln(1.02) = 0.019803

$\Delta = e^{-r_{PD} \cdot T} \cdot [N(d_1) - 1]$ = $e^{-0.019803}$ · (0.6178 - 1) = 0.374706.

d)

d1)

The hedge ratio (HR) is the ratio of the size of the position taken in futures contracts to the size of the exposure. In our case, where the managed portfolio is not neutral with respect to the benchmark ($\beta \neq 1$), the HR is:

$$HR = \beta \cdot \frac{I_0}{F_0} = 1.2 \cdot \frac{4100}{4{,}120} = 1.19417 \approx 1.19$$

d2)

The number of futures contracts to consider is:

$$N_F = -HR \cdot \frac{\text{Portfolio Value}}{\text{Futures Contract Size} \cdot \text{Spot Price}}$$

$$= -1.19417 \cdot \frac{175 \cdot 10^6}{4{,}100 \cdot 10} = -5{,}097.09 \approx -5{,}097$$

Therefore, the strategy requires to short 5,097 futures contacts.