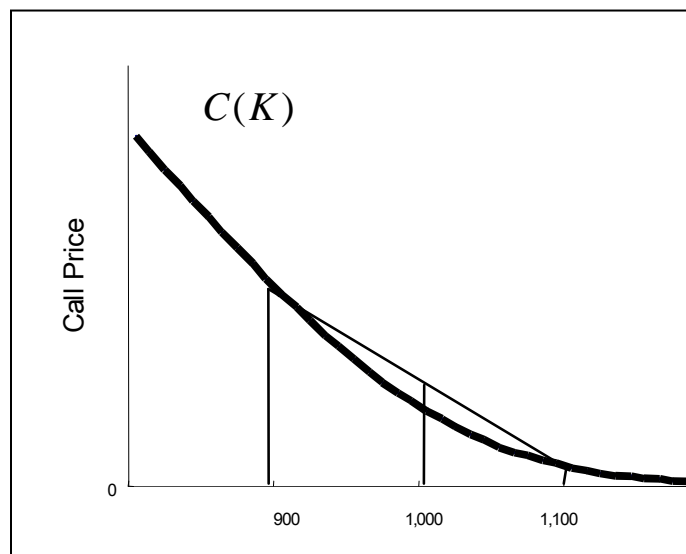


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**Question 5: Derivatives and Portfolio Management****(33 points)**

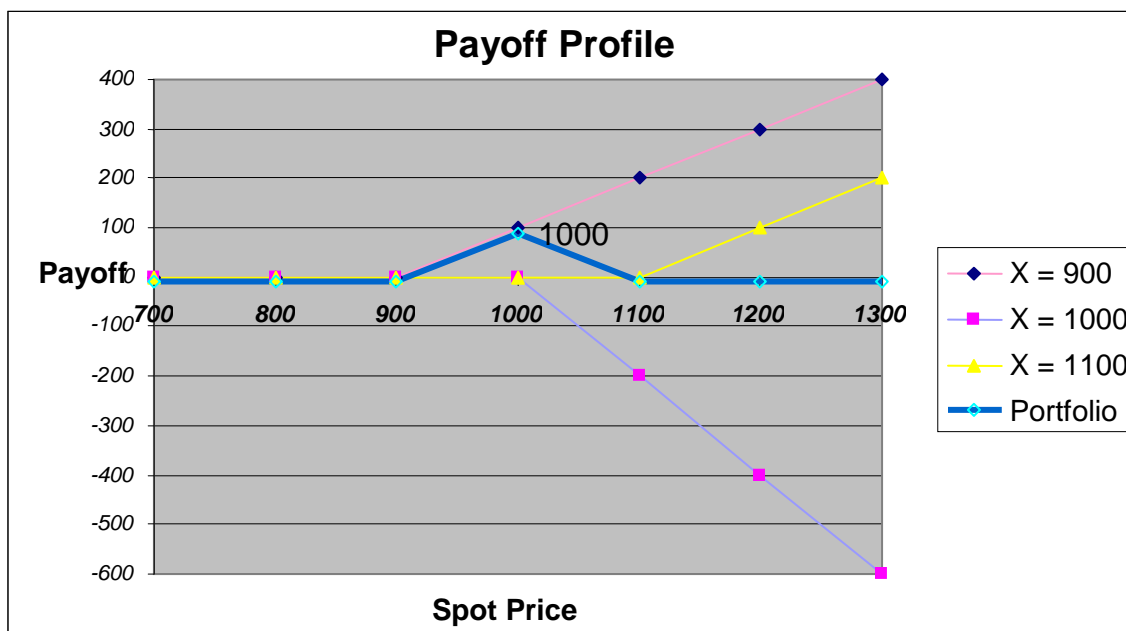
a)

a1)  $C(K)$  represents the price of a call with strike price  $K$ . As can be seen from the chart below,  $C(K)$  is convex and  $C(1000) < \frac{C(900) + C(1100)}{2}$ . Therefore, the premium is higher for the purchased calls ( $C(900) + C(1100)$ ) than the written calls ( $2 \cdot C(1000)$ ), so overall you will pay a premium for the position.



Another way of seeing it is that the Call with strike 900 has intrinsic value 100 plus some time value, whereas the Calls with strike 1000 and 1100 have only time value. Therefore the cost of  $C(900) + C(1100)$  should be higher than  $2 \cdot C(1000)$ .

a2) Below is the payoff at expiration.



a3) Your view is that the stock market will not move much. It will remain around 1000. You also want to limit losses if your view turns out to be wrong. You also want to be neutral to changes in the volatility. Since you are not sure whether the volatility of the underlying will increase or decrease, this position will help you being neutral to changes in volatility in either direction.

b)

b1) With the SMI at 1,000, the delta for the written calls ( $K = 1,000$ ) is almost the same as the deltas for the purchased calls ( $K = 900$  and  $K = 1,100$ ). The total delta is therefore almost 0, so there is no need to buy futures.

b2) As the SMI rises from 1000, the delta of the combined position starts to fall below zero. Hence, the trader will have to buy futures contracts to hedge his position. The delta will keep falling till the SMI reaches 1100. So the trader will have to keep buying futures contracts to dynamically delta-hedge his position. Once the SMI crosses 1100, the delta starts increasing gradually, so the trader needs to gradually sell back the futures. Once the SMI reaches 1200, the delta is around 0, so the trader needs to sell almost all the futures. Beyond 1200, the delta remains almost constant at 0, so the trader does not have to either buy futures or sell futures.

c)

You should write 2 puts with strike prices of 1,000 and buy 1 put each at strike prices of 900 and 1,100, as shown in the formula below.

$$\begin{aligned}
 & C(900) + C(1,100) - 2 \times C(1,000) \\
 &= (P(900) + e^{-rT} \times 900 - S) + (P(1,100) + e^{-rT} \times 1,100 - S) - 2 \times (P(1,000) + e^{-rT} \times 1,000 - S) \\
 &= P(900) + P(1,100) - 2 \times P(1,000)
 \end{aligned}$$

d)

Fund managers decide whether to hedge their portfolios by using either futures contracts or option contracts based on the following factors:

- i. **Cost of Hedging:** The cost involved in using options for hedging an equity portfolio is always higher than the cost involved in using futures. In fact in case of futures, there is no cost involved. Only a margin has to be maintained. In case of options, the cost is the premium paid towards buying the option which is lost irrespective of whether the option position makes a profit or loss.
- ii. **Downside risk:** Options can be used to provide downside risk protection. Futures can be used in a limited way for this purpose, in the form of dynamic insurance using synthetic options.
- iii. **Hedging using beta:** The fund manager can decrease (increase) the  $\beta$  by using futures on the market index and a total immunization can be obtained by bringing the  $\beta$  to zero. However, the same cannot be achieved using Index options since, the  $\beta$  of an option is not constant and varies over time and hence the  $\beta$  has to be re-balanced dynamically.
- iv. **Liquidity of contracts:** The fund manager has to choose between futures or options depending on the availability of the contracts in the market and the liquidity of these contracts. Sometimes, option on the underlying may not exist. For example, in Switzerland, listed puts do not exist for all securities.
- v. **Effect of volatility on Traded Options verses Synthetic options (using futures):** Creating put options on the index synthetically using index futures does not work well compared to using traded put options, if the volatility of the index changes rapidly or if the index exhibits large jumps.