Question 2: Derivatives Analysis and Valuation **(29 points)**

The Nikkei average is currently at 16,000 points. The market trades Nikkei average futures and Nikkei average options (European) which will mature 3 months from now. The futures price is equivalent to the non-arbitrage price; the option price is given in the table below.

	Strike price	Option price	Delta
Call	18,000 JPY	109.820 JPY	0.141
	17,000 JPY	297.563 JPY	0.306
	16,000 JPY	677.146 JPY	0.540
Put	16,000 JPY	P JPY	-0.460
	15,000 JPY	222.369 JPY	-0.228
	14,000 JPY	56.488 JPY	-0.076

Assume that the short-term risk-free interest-rate will remain at a constant 2% p.a. (simple) over the next 3 months. Also, ignore dividends and trading costs.

a) Calculate the current Nikkei average futures price. (3 points)

b) Using the put/call parity equation, calculate the present price P of the Nikkei average put option with a strike price of 16,000 JPY. Round your answer to the 3rd decimal point.

(5 points)

c) You decide to synthesize the payoff at maturity of a unit of Nikkei average put option with a strike price of 16,000 JPY, using a Nikkei average call option with a strike price of 16,000 JPY, borrowing and lending at the short-term interest rate and the Nikkei average futures. At the current point in time, how many units of call options should you buy or sell, what amount of borrowing or lending should you undertake, and what position should you take in the Nikkei average futures? (5 points)

Assume that the fund manager of an equity portfolio worth the equivalent of 1 million times the Nikkei average at all times will be trading futures and options for fund-management purposes. Note that one trading unit for futures and options is 1,000 times the Nikkei average. (Therefore, if the fund manager wishes to purchase 1 trading unit of put options with a strike price of 14,000 JPY, the cost will be $1{,}000 \cdot 56.488$ JPY = 56,488 JPY)

d) The fund manager decides at the current point in time to sell 1,000 trading units of call options with a strike price of 17,000 JPY. Proceeds from the sale of options will be invested at the risk-free rate. If the fund manager maintains this position without

change until the options expire, what will be the profit and loss of the entire position, including the equity portfolio position, option payoff and risk-free assets? For your answer, use a graph showing the value of the Nikkei average on the horizontal axis and the overall profit and loss on the vertical axis. Clearly show the maximum profit value, the break-even level of the Nikkei average and the horizontal values that correspond to the graph's kink points, if any. Round your answers to the integer in million JPY. (6 points)

e) Assume that the position described in d) can be dynamically achieved using the Nikkei average futures rather than by selling call options. To do so, what kind of a position in how many units of futures should you take at the current point in time? Provide numbers and explain your reasoning. Round your answer to the 1st decimal point. (5 points)

f) You take the futures position found in e), and the next instant the Nikkei average drops. How should you change your futures position if you want to maintain the dynamic hedge, i.e. would you buy more futures, or sell more futures? What approach would you take to choose the size of your position? How would you adjust your position over time? Explain your reasons. There is no need to provide actual numbers.(5 points)