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**Question 2: Derivatives Analysis and Valuation****(29 points)**

In the answers below,  $r_f$  stands for the risk-free rate,  $F_0$  for the current Nikkei average futures price,  $S_0$  for the current value of the Nikkei average (= 16,000 JPY),  $C_0(K)$  for the current price of a call option on the Nikkei average with a strike price of  $K$ , and  $P_0(K)$  for the current price of a put option on the Nikkei average with a strike price of  $K$

a)

Dividends and trading costs can be ignored, so the non-arbitrage condition gives the price of a Nikkei average futures price  $F_0$  as:

$$\text{Futures price } (F_0) = \text{Current value } (S_0) \cdot [1 + (r_f \cdot 0.25)] = 16,000 \cdot 1.005 = 16,080 \text{ JPY}$$

b)

$$P_0(K) = C_0(K) - S_0 + \frac{K}{1 + r_f \cdot \tau}$$

$$\Rightarrow P_0(K) = 677.146 - 16,000 + [16,000 / (1 + 0.02 \cdot 0.25)] = 597.544$$

Therefore, the current price of the put option is 597.544 JPY.

c)

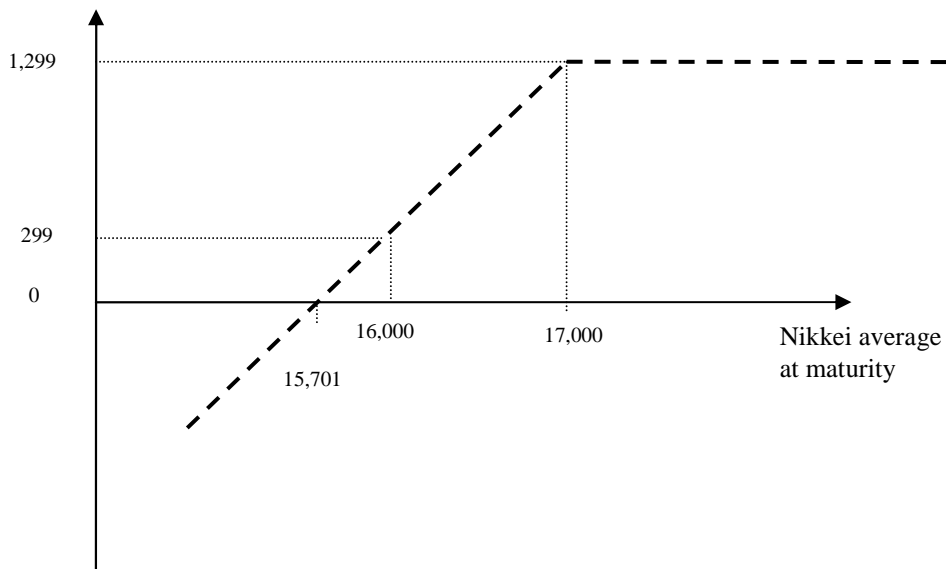
Since the put-call parity can also be written as  $P_0(K) = C_0(K) - \frac{F_0 - K}{1 + r_f \cdot \tau}$  (where  $F$  is the

futures price), to make the synthetic position you have to purchase 1 unit of calls, borrow at the risk-free rate “(Futures price - Strike price)/(1 + 0.02 · 0.25) (= (16,080 - 16,000)/1.005)” JPY, and short 1 unit of the futures.

d)

This is a covered call position on 1,000 trading units using the call option with a strike price of 17,000 JPY. The premium received from the sale of the calls is:  $297.563 \cdot 1,000,000 = 297.563$  million JPY, which invested at 2% for 3 months become:  $297.563 \cdot 1.005 = 299.051$  million JPY.

Therefore, the break-even level of the Nikkei is approximately:  $16,000 - 299 = 15,701$ .  
The maximum profit value is:  $17,000 - 16,000 + 299 = 1,299$  million JPY.



e)

In d), 1,000 trading units of call options with a strike price of 17,000 JPY are sold. These call options have a delta of 0.306, so the delta on the sold position is  $-0.306$ . Meanwhile, in the futures price is equivalent to the arbitrage price and has a delta of 1.005. Therefore, for a dynamic hedge position in which 1 trading unit of call options is sold, you must take a position of “Futures  $- 0.306/1.005 = -0.3045$ ” (a short position in 0.3045 units). What you actually need are 1,000 units, so you multiply  $0.3045 \cdot 1,000$  to find that the futures position to be taken at the current point in time is a short position of 304.5 trading units.

f)

If all of the conditions remain equal, a decline in price for the underlying asset (the Nikkei average) will reduce the call option's delta. The delta on a position in which call options are sold will therefore grow larger. (The absolute value of the negative will be smaller.) Since this is a dynamic hedge, the delta of the futures position must match the new delta of the options position. Therefore, you need to buy back the futures so as to reduce the absolute value of the short position.